Investigation 1

Equations for Circles and Polygons

The photo below shows a "crop circle." Not all crop circles are made in crop fields, nor are they all circles. However, the term "crop circles" is often used to describe all such designs. Designs like these have appeared in fields around the world. At first, the origins of the crop circles were unknown. However, in many cases, the people who made them have come forward and taken credit for their work.

Getting Ready for Problem 1.1

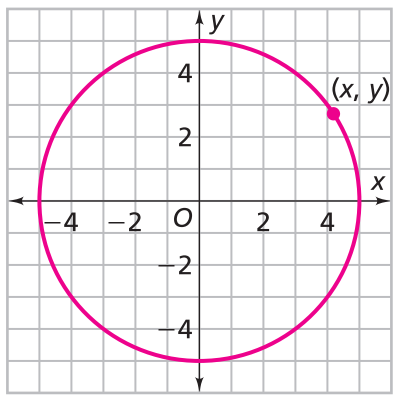
Suppose you are planning to make a crop circle design like the one above.

- How can you outline the circle accurately?
- How can you locate sides and vertices of the other shapes in the design?
- How can you use equations and coordinate graphs to help plan your design?

1.1 Equations for Circles

You can outline the outer circle of a crop circle by using a rope. Anchor one end of the rope where you want the center of the circle. Hold the other end and, with the rope pulled taut, walk around the center point.

To plan the other parts of the design, it helps to draw the circle on a coordinate grid. In this problem, you will find an equation relating the coordinates of the points on a circle.



On the circle above, are there points for which it is easy to find the coordinates?

What mathematical ideas can help you find coordinates of other points on the circle?

Problem 1.1 Equations for Circles

A. 1. The circle above has a radius of 5 units and is centered at the origin. Estimate the missing coordinate for these points on the circle. If there is more than one possible point, give the missing coordinate for each possibility.

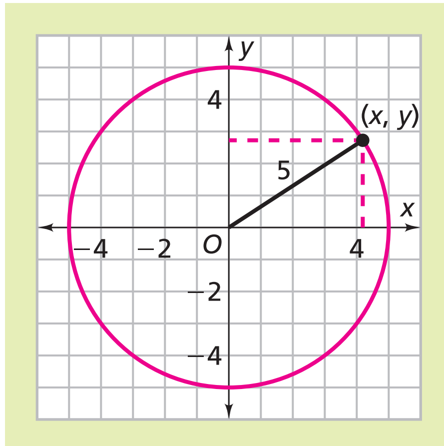
a. (0, ■) **b.** (■, 0) **c.** (3, ■)

d. (4, ■) **e.** (■, −3) **f.** (■, 4)

g. (−2, ■) **h.** (■, 2) **i.** (■, 5)

2. Which of your coordinates from part (1) do you think are exactly correct? How do you know?

B. Think about a point (x, y) starting at $(5, 0)$ and moving counterclockwise, tracing around the circle.



1. How does the y-coordinate of the point change as the x-coordinate approaches zero and then becomes negative?

2. The radius from the origin $(0, 0)$ to the point (x, y) has a length of 5 units. The diagram shows that you can make two right triangles with the radius as the hypotenuse. How do these triangles change as the point moves around the circle?

3. Use what you know about the relationship among the side lengths of a right triangle to write an equation relating x and y to the radius, 5.

4. Kaitlyn says that the relationship is $x + y = 5$ or $y = 5 - x$. Is she correct? Explain.

5. Does every point on the circle satisfy your equation? Explain.

C. These points are all on the circle. Check that they satisfy the equation you wrote in Question B part (3).

1. $(3, 4)$ **2.** $(-4, 3)$ **3.** $(\sqrt{13}, \sqrt{12})$ **4.** $(0, -5)$

5. Does any point *not* on the circle satisfy the equation? Explain.

D. 1. Give the coordinates of three points in the interior of the circle. What can you say about the x- and y-coordinates of points inside the circle?

2. Use your equation from Question B to help you write an *inequality* that describes the points in the interior of the circle.

E. How can you change your equation from Question B to represent a circle with a radius of 1, 3, or 10 units?

ACE **Homework starts on page 12.**